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ANALYSIS OF BINARY FILM BOILING

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Abstract—Binary film boiling has been analyzed on the basis of the conservation laws and other principles. It has been shown that the liquid–vapor interface temperature depends on diffusion phenomena within the liquid mixture. The omission of the diffusion processes in an analysis might lead to erroneous heat-transfer results.

NOMENCLATURE

c_f,	parameter defined by equation (13);
c_g,	parameter defined by equation (4);
c_p,	specific heat;
D,	diffusion coefficient;
f,	dimensionless stream function, vapor;
F,	dimensionless stream function, liquid;
g,	gravitational acceleration;
h_{fg},	latent heat;
k,	thermal conductivity;
$\dot{m}$,	local mass flow rate;
Pr,	Prandtl number;
$\dot{q}$,	local heat flux;
Sc,	Schmidt number;
T,	temperature;
T_s,	interface temperature;
u, v,	velocity components;
w,	mass fraction in liquid;
$\bar{w}$,	mass fraction in vapor;
x, y,	coordinates.

Greek symbols

δ,	local vapor film thickness;
η,	similarity variable;
μ,	viscosity;
ν,	kinematic viscosity;
ρ,	density;
τ,	temperature variable;
ϕ,	mass fraction variable;
ψ,	stream function.

Subscripts

g,	vapor;
f,	liquid;
k,	without diffusion;
s,	interface;
w,	wall surface, vapor;
∞,	liquid bulk;
δ,	interface;
1,	methanol;
2,	water.

INTRODUCTION

A RATHER extensive review of film boiling has recently been provided by Kalinin *et al.* [1]. It is interesting to note that while film boiling of pure liquids has received detailed attention, film boiling of liquid mixtures has virtually been ignored. The only experimental research on film boiling of miscible binary mixtures of volatile components known to the present authors has been carried out by Westwater *et al.* [2, 3] and by Van Stralen *et al.* [4]. These studies indicate that mass diffusion plays an important role in the heat-transfer processes. An attempt to analyze film boiling of binary mixtures including diffusion effects in the liquid was recently made by Marschall [5]. In this analysis it was demonstrated to what extent the diffusion processes in the liquid control the vapor–liquid interface temperature. In contrast to film boiling, nucleate boiling of liquid mixtures has been subject of numerous experimental and some theoretical investigations. A survey of binary nucleate boiling prepared by Stephan and Körner can be found in [6].

The present research is concerned with the formulation and application of a predictive theory for laminar film boiling of a binary liquid mixture. The theory is based on the conservation laws alone, however, it incorporates most major restrictions made by Koh in his analysis of laminar film boiling of a pure liquid [7]. Thus, the following theory may be viewed as an extension of Koh's work.

The physical situation chosen for study is laminar film boiling on a heated, isothermal vertical plate. The plate is situated in a space that is filled with a binary mixture of miscible liquids, both components of which can be evaporated in the temperature range of interest. The vapor film, formed adjacent to the plate, flows upward along the plate under the influence of buoyant forces. It is assumed that the liquid–vapor interface is smooth and free of waves, although a stability analysis undertaken by Kao *et al.* [8] indicates that this situation is quite unlikely. However, it is anticipated that, as demonstrated in laminar film condensation, the assumption of a waveless interface will not lead to large errors.

The bulk of the liquid is assumed to be at a saturation state corresponding to the ambient temperature T_∞ and the total pressure. The interface between liquid and vapor phase is also assumed to be at a saturation state corresponding to the interface temperature T_s and the total pressure. The value of the interface temperature T_s is unknown *a priori* and has to be obtained as a result of the analysis. The temperature T_w of the plate is assumed to take on values high enough to assure uninterrupted film boiling. At a given total pressure, bulk concentration of the liquid mixture and interfacial liquid and vapor concentrations are definitely fixed by the bulk and interface temperatures.

The analytical model employed here treats vapor and liquid flow as incompressible boundary layer flow. Convective motion in the vapor phase is induced by the vapor production at the interface, and by density differences across the vapor film. Convective and diffusive motion in the liquid phase are due to concentration differences within the liquid and due to the fact that the interface acts as a sink for the liquid mixture. Free convection currents in the liquid are neglected. In addition, radiation effects are not accounted for. The error introduced by these assumptions will depend on the wall surface properties as well as the physical properties of the evaporating mixture.

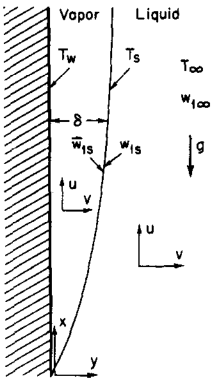

FIG. 1. Schematic of the physical system.

A schematic diagram of the physical situation chosen for study is presented in Fig. 1. Upward and transverse distances are measured in terms of the x-coordinate and y-coordinate, respectively. The thickness of the vapor film is denoted as δ. The thickness varies in direction of the x-coordinate. Also shown in Fig. 1 is some nomenclature for temperatures, concentrations, velocities, and the gravitational acceleration.

The presentation which follows is restricted to binary film boiling. As an example, boiling of a methanol–water mixture will be considered.

ANALYTICAL FORMULATION AND SOLUTION

The mathematical description of the film boiling problem under study requires the separate consideration of the transport processes in the liquid and the vapor phase, along with the appropriate boundary and interface conditions. As indicated before, this description is being done within the framework of laminar boundary layer theory.

Vapor film

The conservation equations for mass, momentum, and energy for the vapor flow are as follows:

Continuity:

$$\frac{\partial u_g}{\partial x}+\frac{\partial v_g}{\partial y}=0 \tag{1}$$

Momentum:

$$u_g\frac{\partial u_g}{\partial x}+v_g\frac{\partial u_g}{\partial y}=g\frac{(\rho_\infty-\rho_g)}{\rho_g}+\nu_g\frac{\partial^2 u_g}{\partial y^2} \tag{2}$$

Energy:

$$u_g\frac{\partial T}{\partial x}+v_g\frac{\partial T}{\partial y}=\frac{k_g}{\rho_g c_{pg}}\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

In this formulation, the physical properties are taken as constants with exception of the vapor density in the buoyancy force term. All constant physical properties are related to the arithmetic average value between the wall temperature T_w and the saturation temperature at the interface T_s and to the appropriate vapor concentrations. The vapor mass fractions are fixed by the interface temperature T_s via equilibrium data. Equations (1)–(3) admit a similarity solution. For this purpose, a new independent variable η_g is defined as

$$\eta_g=c_g y x^{-1/4},\quad c_g=[g(\rho_\infty-\rho_w)/(4\nu_g^2\rho_w)]^{1/4} \tag{4}$$

along with dependent variables

$$f=\psi_g/(4\nu_g c_g x^{3/4});\quad \tau=(T-T_s)/(T_w-T_s) \tag{5}$$

where ψ_g is the stream function, characterized by $u_g=\partial\psi_g/\partial y$ and $v_g=-\partial\psi_g/\partial x$. The similarity transformation yields

$$f'''=2f'^2-3ff''-\frac{\rho_\infty/\rho_g-1}{\rho_\infty/\rho_w-1} \tag{6}$$

$$\tau''+3Pr_g f\tau'=0 \tag{7}$$

in which Pr_g denotes the vapor mixture Prandtl number. An underlying assumption in the derivation of equations (1)–(7) is that any diffusion phenomena in the vapor phase would be a consequence of concentration differences in the vapor mixture alone. Since the vapor–liquid interface is assumed to be at the uniform temperature T_s, the vapor concentrations are uniform, too; and, thus, there are no diffusion processes within the vapor film. Therefore, in the vapor film it is

$$\bar{w}_1;\quad \bar{w}_2=\text{constant}. \tag{8}$$

Liquid boundary layer

In many applications, the temperature changes across the liquid boundary layer are very small so that only a very small fraction of the energy arriving at the liquid–vapor interface is flowing by conduction and convection into the bulk of the liquid. This implies that the omission of the energy equation in the liquid phase will not lead to a considerable error in the analysis. Furthermore, free convection currents in the liquid phase due to concentration and temperature variations across the boundary layer are small and, therefore, will be disregarded. With this, the conservation of mass and momentum is sufficiently described by the following equations:

Continuity:

$$\frac{\partial u_f}{\partial x}+\frac{\partial v_f}{\partial y}=0 \tag{9}$$

Momentum:

$$u_f \frac{\partial u_f}{\partial x} + v_f \frac{\partial u_f}{\partial y} = \nu_f \frac{\partial^2 u_f}{\partial y^2} \tag{10}$$

Conservation of species:

$$u_f \frac{\partial w_1}{\partial x} + v_f \frac{\partial w_1}{\partial y} = D_f \frac{\partial^2 w_1}{\partial y^2}. \tag{11}$$

It is understood that for a binary mixture only one species conservation equation is needed, since the sum of the mass fractions is equal to one

$$w_1 + w_2 = 1. \tag{12}$$

For a similarity transformation of equations (9)–(11), the following independent variable is defined

$$\eta_f = c_f y / x^{1/4}; \quad c_f = \left(\frac{g(\rho_\infty - \rho_w)}{4 \nu_f^2 \rho_w} \right)^{1/4} \tag{13}$$

along with dependent variables

$$F = \psi_f / (4 \nu_f c_f x^{3/4}); \quad \phi = \frac{w_1 - w_{1\infty}}{w_{1s} - w_{1\infty}} \tag{14}$$

where ψ_f is the stream function in the liquid phase. With the foregoing variables, mass and momentum conservation equations are transformed into the following ordinary differential equations

$$F''' = 2F'^2 - 3FF'' \tag{15}$$

and

$$\phi'' + 3Sc_f F \phi' = 0 \tag{16}$$

in which Sc_f is the Schmidt number of the liquid mixture. To complete the mathematical description of the problem under study, boundary and interface conditions have to be specified.

Boundary and interface conditions

At the wall all velocities are equal to zero and the vapor temperature is equal to the uniform wall temperature.

$$y = 0 \qquad \begin{aligned} u_g &= 0 \\ v_g &= 0 \\ T &= T_w \end{aligned} \tag{17}$$

or, in terms of the similarity variables

$$\eta_g = 0 \qquad \begin{aligned} f &= 0 \\ f' &= 0 \\ \tau &= 1. \end{aligned} \tag{18}$$

At the interface, liquid and vapor are at the saturation temperature T_s. At a given pressure the value of T_s determines the mass fraction in both liquid and vapor phase at the interface.

$$y = \delta \qquad \begin{aligned} T &= T_s \\ w_1; \ \bar{w}_1 &= w(T_s). \end{aligned} \tag{19}$$

Assuming non-slip conditions at the interface and applying the principle of conservation of mass yield

$$y = \delta \qquad \begin{aligned} u_g &= u_f \\ \mu_g \frac{\partial u_g}{\partial y} &= \mu_f \frac{\partial u_f}{\partial y} \\ \dot{m}_g &= \dot{m}_f \\ \dot{m}_{1g} &= \dot{m}_{1f}. \end{aligned} \tag{20}$$

An additional condition is obtained from an energy balance at the interface

$$-k_g \left. \frac{\partial T}{\partial y} \right|_{y=\delta} = \rho_g h_{fg} \frac{\partial}{\partial x} \int_0^\delta u_g \, \mathrm{d}y \tag{21}$$

Equation (21) states that all energy conducted across the interface is purely used to evaporate the liquid mixture. Equations (19)–(21) written in terms of the similarity variables are

$$\eta_g = \eta_{g\delta}, \quad F = f \left(\frac{\mu_g \rho_g}{\mu_f \rho_f} \right)^{1/2} \tag{22}$$

$$(\eta_f = 0) \qquad F' = f' \tag{23}$$

$$F'' = f'' \left(\frac{\mu_g \rho_g}{\mu_f \rho_f} \right)^{1/2} \tag{24}$$

$$\tau = 0 \tag{25}$$

$$\phi = 1 \tag{26}$$

$$\phi' = 3F Sc_f \left(\frac{\bar{w}_{1s} - w_{1s}}{w_{1s} - w_1} \right) \tag{27}$$

and

$$\frac{k_g (T_w - T_s)}{h_{fg} \mu_g} = -3 \frac{f}{\tau'}. \tag{28}$$

The physical condition in the bulk of the liquid

$$y = \infty \qquad \begin{aligned} u &= 0 & \frac{\partial u}{\partial y} &= 0 \\ v &= 0 & & \\ T &= T_\infty & \frac{\partial T}{\partial y} &= 0 \end{aligned} \tag{29}$$

when written in terms of the similarity variables, become

$$\eta_f = \infty \qquad F' = 0 \tag{30}$$

$$F'' = 0 \tag{31}$$

$$\phi = 0 \tag{32}$$

$$\phi' = 0. \tag{33}$$

Again, it should be noted, that the liquid bulk concentrations are known if, at a given pressure, the temperature T_∞ is known.

The solution of the preceeding equations provides pertinent information on laminar film boiling heat-transfer characteristics. To obtain the solution a procedure has to be employed which will now be described in detail.

The bulk temperature T_∞ of the saturated liquid mixture and the dimensionless vapor film thickness η_g are specified.

Next, trial values for the wall temperature T_w, the interface temperature T_s, and the interface velocities $u_g = u_f$ are selected. With this all relevant physical properties in the vapor and liquid phase can be determined.

Then, equations (6) and (7) along with the boundary conditions (18), (23) and (25) are solved. Equation (28) is then used to check the trial value of T_w and if necessary, to correct it by an iterative procedure.

The next step is to solve the differential equations (15) and (16) along with the boundary conditions (22), (23), (26), (30) and (32). Boundary conditions (31) and (33) are utilized to ensure that the integration of equations (15) and (16) is being carried out across the entire liquid boundary-layer thickness.

Equations (24) and (27) are now used to check the validity of the assumed values of T_s and $u_g = u_f$. An iterative procedure is used to correct the values of T_s and $u_g = u_f$ until equations (24) and (27) are satisfied.

The so found values of T_w, T_s and $u_g = u_f$ are now used as a new set of trial values and the just described calculations are repeated until two consecutive sets of values for the wall temperature, interface temperature and interface velocity agree with each other.

As final results, velocity and concentration fields in liquid and vapor phase and the temperature field in the vapor film are obtained. Of foremost interest are the heat transfer results. The local heat flux $\dot{q}$ is found from:

$$\dot{q} = -k_g(T_w - T_s)\tau'_w c_g x^{-1/4} \tag{34}$$

while the local Nusselt number can be evaluated from:

$$Nu = -\tau'_w c_g x^{3/4}. \tag{35}$$

FILM-BOILING OF METHANOL-WATER MIXTURES

The foregoing analytical formulation and solution method was applied to the film-boiling of methanol-water mixtures at atmospheric pressure. Physical properties of water and methanol as well as rules to determine mixture properties for liquid and vapor phase are well known and can be found in [9–14]. Solutions were carried out for values of T_∞ in the range of 67–97°C and for values of $T_w - T_s$ ranging from 400 to 800°C.

Figure 2 shows the first derivative τ'_w of the dimensionless temperature function τ at the wall as a function of the temperature difference $T_w - T_s$ for various temperatures T_∞. The temperature range of T_∞ is chosen such that the concentrations in the liquid phase almost cover the range from pure methanol to pure water. According to equations (34) and (35) the local heat flux and the local Nusselt number are related to τ'_w by the following equations

$$-\tau'_w = \frac{\dot{q}x^{1/4}}{k_g(T_w - T_s)c_g} \tag{36}$$

$$= \frac{Nu}{x^{3/4}c_g}. \tag{37}$$

An evaluation of the local heat flux and the local Nusselt number for a fixed location x and a fixed T_∞ value shows that the heat flux $\dot{q}$ is increasing while the local Nusselt number Nu is decreasing with an increasing temperature difference $T_w - T_s$. This behavior is well known for film boiling of pure liquids, thus, at least in that respect, the characteristics of film boiling of pure liquids and binary liquid mixtures are very similar.

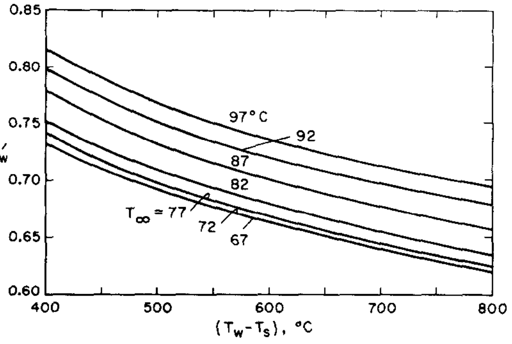

FIG. 2. Heat-transfer results.

Further inspection reveals that at a fixed location and a fixed temperature difference $T_w - T_s$ local heat flux as well as local Nusselt number increase with increasing water concentration in the liquid water methanol mixture. This increase is, however, not linear due to the behavior of the physical properties of the vapor mixture.

Figure 3 shows the difference between interface temperature and liquid bulk temperature $T_s - T_\infty$ for various liquid bulk concentrations. For any fixed value of $T_w - T_s$ the temperature difference $T_s - T_\infty$ is zero for pure water and pure methanol. Starting with pure methanol, the difference $T_s - T_\infty$ increases with increasing water concentration in the liquid mixture, reaches a maximum value in a bulk concentration range corresponding to a saturation temperature range from T_∞ = 82°C to T_∞ = 87°C, and then decreases with increasing water concentration until it disappears for pure water.

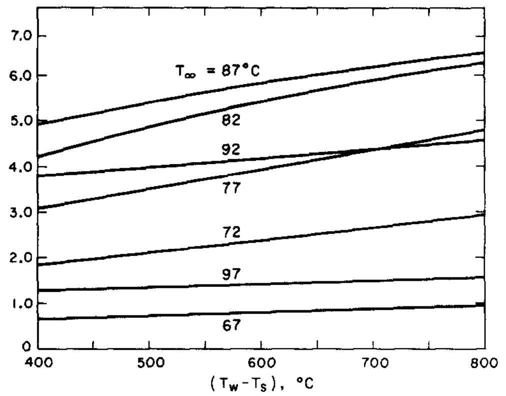

FIG. 3. Difference between interface and liquid bulk temperature as a function of liquid bulk concentration and difference between wall and interface temperature.

The temperature difference $T_s - T_\infty$ is very small as compared to the difference $T_w - T_s$. Consequently, the arising of the temperature difference $T_s - T_\infty$ due to diffusion currents within the liquid mixture should have a negligible influence on the heat transfer. The reason that in a calculation the omission of the diffusion phenomena might lead to erroneous results lies in the interdependence of the interface temperature T_s and the physical properties of the vapor mixture. To demonstrate this, film boiling of a methanol-water mixture at atmospheric pressure has been evaluated for a wall temperature of 700°C. The calculation was carried out once including the diffusion phenomena in the liquid phase as described earlier, and once neglecting the diffusion in the liquid by specifying that

$$T_s = T_\infty. \tag{38}$$

The second case would be equivalent to applying Koh's theory for film boiling of a pure liquid to a binary film boiling problem. In Fig. 4, Prandtl numbers of the vapor mixture, dimensionless local heat fluxes at the wall and the temperature difference $T_s - T_\infty$ are plotted against the liquid bulk temperature T_∞. The difference $T_s - T_\infty$ is zero for the case when the liquid diffusion phenomenon has been neglected. The Prandtl number Pr_g was calculated including the diffusion while the Prandtl number Pr_{gk} was evaluated without considering diffusion on the liquid phase. The differences in the Prandtl numbers point to the fact that the physical properties in the vapor phase depend largely on the interface temperature T_s. The difference between the dimensionless heat flux τ'_w, calculated including diffusion, and the dimensionless heat flux τ'_{wk}, evaluated omitting diffusion are also a consequence of the different interface temperature in the two considered cases. Although the disagreement between τ'_w and τ'_{wk} is not large for the considered example, it might well be of unacceptable magnitude for different mixtures.

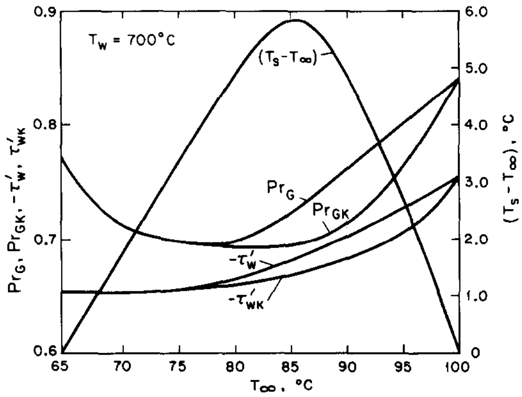

FIG. 4. Comparison of results obtained with and without consideration of diffusion.

CONCLUSION

This study demonstrates that diffusion phenomena in the liquid phase have an influence on the film boiling characteristics. The main reason for this influence is that the physical properties of the vapor mixture depend on the liquid–vapor interface temperature which in turn depends on the diffusion in the liquid phase. While this influence might be relatively small, as shown in the presented example, this influence might be rather large for different mixtures.

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ANALYSE DE L'EBULLITION EN FILM BINAIRE

Résumé—On étudie l'ébullition en film binaire à partir des lois de conservation et d'autres principes. On montre que la température de l'interface liquide–vapeur dépend du phénomène de diffusion dans le mélange liquide. L'omission du processus de diffusion dans l'analyse peut conduire à des résultats erronés sur le transfert thermique.

UNTERSUCHUNG DES FILMSIEDENS BINÄRER GEMISCHE

Zusammenfassung—Auf der Grundlage der Erhaltungsgesetze und anderer Prinzipien wurde das Filmsieden binärer Gemische untersucht. Es wurde gezeigt, daß die Temperatur der Flüssigkeits-Dampf-Phasengrenzfläche von Diffusionsvorgängen innerhalb der Flüssigkeit abhängt. Eine Nichtbeachtung der Diffusionsprozesse könnte zu fehlerhaften Wärmeübergangsergebnissen führen.

АНАЛИЗ КИПЕНИЯ БИНАРНОЙ ПЛЕНКИ

Аннотация — На основе законов сохранения и других принципов анализируется кипение бинарной пленки. Показано, что температура поверхности раздела жидкость-пар зависит от диффузии в жидкой смеси. Пренебрежение диффузионными процессами при расчете теплопереноса может привести к ошибочным результатам.